Keywords: Blade, vibration, damping, rubber, thermo-mechanical characteristics.

Abstract. The proposed contribution deals with a novelty research of a bladed wheel vibration suppression by inserted elements of temperature resistive rubberlike materials. The FKM rubber was selected as a perspective material since it is known by its resistances against high temperatures and against aggressive chemicals. As a study case the five-blade bundle linked by wedge-form rubber elements imposed in the head interspaces of the neighboring blades was chosen. Thermo-mechanical effect on the dynamics of the bundle was investigated numerically and experimentally and a good agreement was achieved in modal characteristics of both approaches. The results of the study show that although the damping markedly decreases in high temperature 150°C with respect to a room temperature, the damping capacity of the FKM elements remains still relatively high with comparison to the inter-blade structural damping by means of the dry friction.
1 INTRODUCTION

Although the bladed wheels of turbo-machines are carefully designed, it is not possible to omit a danger resonant vibration during variant service conditions. Since material damping of the wheel is very light, the construction damping with higher dissipation of mechanical energy helps to diminish the vibration amplitudes and to prolong the life-time of the wheel. Till now we have dealt with a classical solution of the construction damping by dry friction contacts in shrouds by different means, e.g. imposed friction elements, direct blade contacts or tie wire, in our works e.g. [1-3].

The proposed contribution, however, deals with a novelty research of a bladed wheel vibration suppression by imposed elements of rubberlike temperature resistive materials. The FKM rubber was selected as a perspective material since it is known by its resistances against high temperatures and against aggressive chemicals. It makes a good presumption for utilization of such a material in hard operational conditions even such as in low pressure steam turbines.

Since the FKM rubber as other rubber materials poses very strong temperature dependence of material behaviour, such as the Young modulus, loss factor, hardness, we dealt first with the thermo-mechanical description of its material parameters. On the one hand we use the analytical formula ascertained by fitting the tabular material data of the VITON from the literature [4] and on the other hand we use experimental modal analysis of the five-blade bundle under different temperatures. The first case was used for the estimation of the rubber behaviour under high temperatures where the material constants are almost constant and the other case was used for the transient zone temperatures where the material constants' changes are very steep and the tabular values can change substantially from the actual ones. The material constants for the FKM rubber could be directly obtained by the analytical formula, however, in case of modal analysis we should identify them also indirectly by their tuning in the numerical model to fit the experimental results.

As a study case the blade bundle of five blades linked by wedge-form rubber elements imposed in the head interspaces of the neighbouring blades was chosen. Three-dimensional finite element model (FEM) of the blade bundle with imposed rubber elements was created. As shown in our research of the rubber behaviour [5], the dissipation stress tensor expressed by linear function of strain velocity deviator tensor suits very well for description of the rubber damping under small strains. Since it corresponds to the special case of proportional damping model, where damping matrix is proportional only to stiffness matrix, we use this model for the material damping of the rubber.

In the contribution, the model of the blade bundle and evaluation of the rubber material constants are described and the influence of the rubber elements on the dynamics and especially damping of the blade bundle are discussed with respect to a temperature effect.

2 THERMO-MECHANICAL MODELING OF FKM RUBBER BEHAVIOUR

2.1 Analytical formula for FKM

The rubber FKM with a trade mark VITON of hardness 70 ShA has a high resistance against temperatures (up to $220^\circ$C), resistance against majority of aggressive chemicals, synthetic and mineral oils, sunbeams and ozone. Therefore it is used in many industrial applications, e.g. seals and bearings, requiring maximum resistance to high temperatures [6]. This material similarly as many polymers has very strong dependence of its mechanical behavior on the temperature and frequency. It is manifested especially in the so-called transition zone, i.e. temperatures between the glassy and rubbery state of the elastomer. The dependences of
the FKM complex Young modulus $E^* = E_{Re} + iE_{Im}$ is described by the analytical formula in [4]:

$$E^* = 10.49 \times (1 + 0.1i) + 15.88 \times (1 + 0.06i)/((if\alpha T)^{0.64} / (1 + 0.0063( if\alpha T)^{0.64})$$

(1)

where $\alpha T = 10^{-6}(-20(T - 4.4))/(134.5 + T)$.

This formula is function of so-called "reduced frequency" $f_r = f\alpha(T)$ where $\alpha(T)$ is the frequency shift factor due to temperature and is modeled by the Williams-Landel-Ferry equation. Temperature dependences of the Young modulus $E$ and loss factor $\tan(\delta) = E_{Im}/E_{Re}$ of the VITON rubber for frequencies 100Hz and 1000Hz are depicted in Fig.1. Transition zone is situated in temperature range from (-10,40)$^\circ$C for the frequency range.

![Figure 1: Young modulus (top) and loss factor (bottom) characteristics versus temperature for the frequencies 100Hz (solid) and 1000Hz (dash line).](image_url)

2.2 Experimental modal analysis of the bundle

The modal analysis was prepared in order to study experimentally the FKM material behaviour changes due to the temperature and to evaluate temperature influence on the dynamics of the five-blade bundle with imposed FKM elements. The elements were squeezed into the blade head interspaces to create elastic connections between the rubbers and the blades. For simulation of the temperature changes of the rubber, the elements and the blade heads were heated up to 50$^\circ$C before the tests. Modal analysis were performed afterwards for several temperatures during the cooling process. The temperatures of the rubber and the heads was measured by the IR thermometer.

The modal analysis of the five-bladed bundle was performed by use of the measurement system PULSE 11.0. Three electromagnets EM1, EM2 and EM3 placed against the heads of
the blade B1, B3 and B5 (Fig.2) were installed for the optimal eigenmode excitation. The coils of electromagnets with a ferrite core were supplied by the high power amplifier EP4000. The forced transducers LUKAS S22 placed under the electromagnets measured their force excitation. For measurement of axial accelerations the miniature one-dimensional accelerometers B&K4374 fixed on the blades B1, B3 and B5 denoted as A1, A3 and A5, respectively, on the opposite sides of the blades than the electromagnets. The responses A1, A3 and A5 were picked up simultaneously. Frequency response functions (FRF) of the blades B1, B3 and B5 with respect to a selected electromagnet were evaluated in a range (200, 400)Hz with a resolution 0.125Hz. For the FRF’s evaluation the swept sine excitation in a range (200, 400)Hz and sweep rate 4Hz/s was applied. Force signal was generated by the system PULSE and amplitudes of electromagnet swept sine forces achieved values 0.6N. The first flexural eigenvalues and their corresponding eigenmodes were identified by the LSFD method [7].

Figure 2: Experimental set-up of the five-blade bundle modal analysis.

The results of the first three eigenvalues in the form of eigenfrequencies and damping ratios are displayed for different temperatures both in tabular (Tab.1) and graphic form (Fig.3). The results show that the first eigenfrequency does not change with temperature and the damping ratio is very low for all tested temperatures. The next eigenfrequencies and damping ratios decrease with growing temperature qualitatively similar like Young modulus and loss factor (see Fig.1). Their absolute values are, contrary to the material constants, dependent on the parameters of the mechanical system and on its vibration mode. For the first eigenmode, all blades move in-phase and there is a very small deformation of the rubber elements and
hence a small vibration damping. The higher eigenmodes the more out-of-phase movements
of the blades and therefore higher deformation of the rubber elements. It causes an increase
of damping ratios and also eigenfrequencies since the stiffness of the rubber elements plays
much more important role in the dynamics of the bundle.

<table>
<thead>
<tr>
<th>Eigenvalues</th>
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<tbody>
<tr>
<td>T [°C]</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>50</td>
</tr>
</tbody>
</table>

Table 1: The damping ratios and eigenfrequencies of the first-three eigenvalues of the five-blade bundle for different temperatures.

![Figure 3: Damping ratio (left) and eigenfrequency (right) temperature characteristics of the five blade bundle.](image)

3 NUMERICAL MODAL ANALYSIS OF FIVE-BLADE BUNDLE WITH IMPOSED RUBBER ELEMENTS

The spatial finite element model (Fig.4) was created and computed in the ANSYS software
for the five-blade bundle modal analysis. The steel and rubber parts were modeled by the hex-
agonal elements SOLID45. The nodes of blade and rubber elements on the contact interfaces
were merged to simulate nonslip elastic coupling. The blades were clamped, i.e. zeros dis-
placements were prescribed, in the selected nodes of the bottom parts of the model. Steel con-
stants of the blades were: E = 2.1.10¹¹ Pa, ρ = 7830 kgm⁻³, µ = 0.3. The material constants of
the rubber: ρᵣ = 1850 kgm⁻³. Poisson’s constant µᵣ = 0.49, the Young modulus and damping
coefficient were ascertained by tuning computed eigenfrequencies and damping ratios accord-
ing to the experimental modal analysis. It was approved by the simulations that values of
damping coefficient have not significant influence on the size of the selected eigenfrequencies
so that the value of the Young modulus can be tuned according to the values of eigenfrequen-
cies independently on it.
The equation of motion for free vibration of non-conservative numerical model can be written in matrix form as

$$\mathbf{Kx}(t) + \mathbf{Bx}(t) + \mathbf{Mx}(t) = 0$$

(2)

where stiffness $\mathbf{K}$, damping $\mathbf{B}$ and mass $\mathbf{M}$ matrices are symmetric matrices, $\mathbf{x}, \dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ are vectors of displacement, velocity and acceleration, respectively.

Equation (1) should be transformed for modal analysis into generalized eigenvalue problem

$$\begin{pmatrix} \mathbf{K} + \lambda_i \mathbf{B} + \lambda_i^2 \mathbf{M} \end{pmatrix} \mathbf{x}_i = 0,$$

(3)

where $\lambda_i$ represents set of eigenvalues and $\mathbf{x}_i$ are their corresponding eigenvectors. The $i^{th}$ eigenvalue can be expressed as $\lambda_i = -\zeta_i \omega_i \pm j \omega_i$, where $\zeta_i$ is a damping ratio, $\omega_i$ an eigen-frequency. So, we assumed that the differences between eigenfrequencies of the damped and the undamped system are negligible for the calculation of damping ratios.

The damping matrix $\mathbf{B}_r$ of the rubber model was described by special form of the proportional damping as

$$\mathbf{B}_r = \beta \mathbf{K}_r,$$

(4)

where $\beta$ is a stiffness coefficient of the damping and $\mathbf{K}_r$ is a stiffness matrix of rubber elements. The relation between a loss factor and a stiffness coefficient can be expressed as $\tan(\delta) = \beta \omega$, where $\omega$ is a vibration frequency. The local damping matrices $\mathbf{B}_r$ of rubber elements contribute only to the global damping matrix $\mathbf{B}$ and hence matrix $\mathbf{B}$ is not proportional to stiffness matrix $\mathbf{K}$. It causes that the solution of the generalized eigenvalue problem

Figure 4: FE model of the blade bundle (brown) with inserted rubber elements (blue) - oblique view.
(3) leads to the complex eigenvectors despite that the proportional damping is used. The Lanczos method specified for damped structures was opted for the solution of this problem.

3.1 Results

The numerical modal analysis of the blade bundle computed under different temperatures showed that a temperature has a significant effect on the computed eigenfrequencies and damping ratios but has negligible effect on the forms of eigenmodes in the investigated temperature range. The numerical results of the bundle eigenvalues for three different temperatures, i.e. 150°C, 50°C and 150°C, are displayed in Tab.2-4, respectively. Material constants of the rubber ascertained for the each case are shown here, too. The experimental counterparts to the computed eigenvalues are associated only for the temperature 150°C, 50°C where the experimental results are available. The rubber constants for the temperature 150°C were enumerated by means of the mentioned analytical formula. As it was already mentioned the damping arises with the deformation of the elements, and then stiffness of the elements more influences the dynamical coupling between the blades. It can be seen from the computed eigenmodes (Fig. 5) that with a higher order number \( i \) of the mode the higher is the number of nodal lines inside the bundle causing the higher distortion of the rubber elements. The nodal lines are here equivalent to nodal diameters of a bladed disc.

The comparison of the numerical and experimental eigenvalues show that the numerical model describes good the temperature effect on the rubber elements in the temperatures up to 50°C. Therefore we assumed that it can be used for assessment of the bundle behavior under high temperatures, e.g. 150°C, where the Young modulus and loss factor are quite unchangeable (Fig.1). Although the loss factor falls to the values around 0.1 and the damping mark-
edly decreases with respect to a room temperature, the damping capacity of the rubber elements in the bundle remains still relatively high (2.8%) with comparison to the other ordinary type of the interblade structural damping, e.g. by means of the dry friction. Dry friction damping effect is also dependent on the relative motions of the blades but achieves according to our experiences maximal values around 1-2%.

<table>
<thead>
<tr>
<th>FEM</th>
<th>EXPERIMENT</th>
</tr>
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<tbody>
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<td>E=150MPa, ρ=1850kgm⁻³, β=4.5e⁻⁴ T=15°C</td>
<td>ζ [%]</td>
</tr>
<tr>
<td>1</td>
<td>0.01</td>
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<tr>
<td>2</td>
<td>9.14</td>
</tr>
<tr>
<td>3</td>
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Table 2: The numerical and experimental damping ratios and eigenfrequencies of the first-three eigenvalues of the five-blade bundle for temperature T=15°C.

<table>
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</tr>
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<tr>
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<tr>
<td>2</td>
<td>2.18</td>
</tr>
<tr>
<td>3</td>
<td>6.29</td>
</tr>
<tr>
<td>4</td>
<td>10.13</td>
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</table>

Table 3: The numerical and experimental damping ratios and eigenfrequencies of the first-four eigenvalues of the five-blade bundle for temperature T=50°C.

<table>
<thead>
<tr>
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<th>T=150°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>E=15MPa, ρ=1850kgm⁻³, β=8e⁻⁵</td>
<td></td>
</tr>
<tr>
<td>i</td>
<td>ζᵢ [%]</td>
</tr>
<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
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<tr>
<td>3</td>
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</tr>
<tr>
<td>4</td>
<td>2.18</td>
</tr>
<tr>
<td>5</td>
<td>2.82</td>
</tr>
</tbody>
</table>

Table 4: The numerical and experimental damping ratios and eigenfrequencies of the first-five eigenvalues of the five-blade bundle for temperature T=150°C.

4 CONCLUSIONS

• Numerical FE model of five-blade bundle with imposed damping elements made of special rubber FKM has been created and applied for the ascertaining of modal behaviour of blades bundle. The special case of proportional damping model was utilized for the material damping of the rubber.

• Analysis of temperature effect on this material showed that the Young modulus and loss factor varies mainly in the temperature range under 50°C. Above 50°C the influence of temperature is negligible.
• The physical model of the five-blade bundle with the damping elements was composed and its experimental modal analysis for several temperatures was performed.

• Thermo-mechanical effect on the dynamics of the bundle was investigated numerically and experimentally and a good agreement was achieved in modal characteristics of both approaches.

• Effect of the temperature is markedly seen on the eigenvalues, on the forms of eigen-modes it is negligible in the investigated temperature range.

• It was shown that dissipation of the mechanical energy of the bundle depends on the shape deformations of the rubber elements and therefore the damping ratio increases with the number of nodal lines inside the bundle. Nodal lines corresponds here to nodal diameters of a bladed disc.

• Although the loss factor falls to the values around $1.10^{-2}$ for temperatures around $150^\circ C$ and the damping markedly decreases with respect to room temperatures, the damping capacity of the rubber elements remains still relatively high (2.8%) with comparison to the other ordinary type of the interblade structural damping by means of the dry friction. Dry friction damping effect is also dependent on the relative motions of the blades but achieves according to our experiences maximal values around 1-2%.

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